CHAPTER 0

The Advantage of Abstraction

A. F. J. Levi, January, 2022

Introduction

Before spending precious time and effort learning something new it is sometimes worthwhile considering the big picture. Understanding and even contributing to knowledge almost inevitably builds on concepts and ideas from the past. With this in mind, the following touches on previous successes of science and technology, its impact on society, and what the future could hold.

We start, as one might have guessed, with the idea of nothing – the number zero.

To zero and beyond!

A practical application of positive integer numbers is counting physical objects. Experiments can be performed in which such real objects are added or subtracted and the result observed as a measurement.

If all the objects are taken away there is nothing to be observed. Clearly, the idea of zero and negative integer numbers is not directly related to physical objects whose presence can be measured. These useful concepts are an *abstraction* – they are different from the actual physical observation experienced.

More than 2,400 years ago the Babylonians counted in base 60 and used a zero symbol when writing numbers, but it was not used as an actual number in the same way it is used today. Similarly, the Ancient Greeks and Romans did not use zero as an actual number.

The first use of the number zero in the modern sense is documented in India in 628 AD. This is important because it requires a level of abstraction that allows representation of a concept that cannot be observed in an experiment that, for example, counts physical objects. Negative numbers, such as the solution to the algebraic expression $4x + 32 = 0$, are likewise an abstraction that allows calculations of practical importance and development of predictive models.

Division and multiplication, including division and multiplication by zero, adds to the ability to develop predictive models. Rational and irrational numbers follow.

In geometry, calculating the area of a square of side a in Euclidean space, where a is a positive number, gives $A = a \times a = a^2$ in which A is a positive number and $a = \sqrt{A}$. However, if the area is a negative number, $-A$, then $a = \mathrm{i}\sqrt{A}$ where $\mathrm{i} = \sqrt{-1}$ so that $-A = \mathrm{i}^2\sqrt{A}\sqrt{A} = -1 \times A = -A$. [1] The pure imaginary number i cannot be observed because the results of physical measurement can only be real numbers.

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[1] Gauss introduced the symbol i for the square root of minus one in 1831.

Notice, for two complex numbers a and b the order in which they are operated on matters so that, for example, $\sqrt{a}\sqrt{b} \neq \sqrt{ab}$ since $\sqrt{-1}\sqrt{-1} = \mathrm{i}^2 = -1$ which is not the same as $\sqrt{-1 \times -1} = 1$.

Different ways to solve problems

The big picture as it relates to expanding knowledge in the physical sciences and engineering focuses on development of predictive theories, experimental validation, and experimental discovery. Historically, and in stark contrast to purely empirical Edisonian methods, the development of *forward physical models* that, via abstraction, describe the relationship between principal parameters determining behavior has played a key role in advancement of science and the ability to efficiently solve practical problems, be they classical or quantum. [2] The process of abstraction to create a physical model involves, by definition, some loss of information about how the system behaves. Importantly, the general applicability of a predictive physical model is inherently more valuable than identification of a single system configuration with desired properties.

Nevertheless, in recent times, *optimization* and its inverse problem subclass has created new applications because it provides a different methodology to solve problems. Optimization seeks via automated search to minimize the distance between the result of a forward solve or measurement and an objective. Inverse methods in the discrete domain, such as satisfiability and other NP (nondeterministic polynomial-time) complete problems, likewise seek solutions to objective functions, often in the presence of constraints.

Part of the success of contemporary *machine learning* (ML) and *artificial intelligence* (AI) is that it has established yet another approach to solving problems of practical importance. ML methods find relationships in data such as classification of features or identification of principal components. This automated extraction of relevant information from a large dataset is an abstraction process that enables a simplified description of systems. Today's *energy inefficient* AI and ML implementations are only possible because of the availability of significant and inexpensive compute resources, itself a result of innovation in hardware and system integration.

AI and ML has been applied to many problems including machine recognition of natural language, autonomous systems, pattern recognition, and image classification in very large datasets. Much of the research in this topic has been performed at the intersection between Electrical Engineering and Computer Science.

However, there remains much to be understood about the fundamentals of AI. For example, there is no agreed *measure* of intelligence in AI. If there were, it would enable a scientific approach to developing new concepts and systems. A measure would exist to quantify intelligence and the distance from that objective. Absent serendipitous Edisonian good luck, it seems reasonable to assume that efficient progress creating intelligent systems could be achieved by adopting such a scientific approach.

Forward physical models in quantum mechanics

Physical models that have contributed so much to the development of science and technology are themselves built as abstractions. The predictions of Newtonian [3] classical mechanics, Hamiltonians, and Lagrangians, come from models of real forces and real

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[2] Abstraction implies simplification and an advantage of developing simple models is they are easier to explain to others.

[3] Newton was born in 1643 and died in 1727.

fields that can be measured and capture the parameters determining key aspects of system behavior. The use of complex numbers and complex functions is a further abstraction that both allows development of different predictive models and opportunities for the design and control of systems. The wave function, the Schrödinger field ψ, in quantum mechanics can be complex and so it is not a quantity that can be measured. This means that the development of predictive forward physical models in quantum mechanics depends on use of imaginary numbers and complex functions that cannot be directly measured!

While the wave function, ψ, evolves deterministically in time according to a wave equation and can be used to describe the behavior of atomic and nano-scale systems, the results of measurement are fundamentally non-causal. Non-relativistic quantum mechanics has other strange attributes. It is linear, so that linear superposition of particle states can be formed, identical indistinguishable particles exist, and there is the concept of particle entanglement. Forward physical models in non-relativistic quantum mechanics use the linear algebra of non-commuting operators and form the basis for understanding physical phenomena on an atomic scale. An electron point particle of rest mass m_0, charge magnitude e, and quantized spin magnitude $\hbar/2$, can behave as a wave. A light wave of radial frequency ω can behave as a quantized particle – the photon. This wave-particle duality of particles such as electrons or photons is unique to quantum mechanics. Particle energy $E = \hbar\omega$ is quantized on a scale set by Planck's constant $\hbar = 1.0545 \times 10^{-34}$ J s.

Part of the value of quantum mechanics is that it has applications in engineering, including semiconductor transistors, lasers, and quantum optics. Because much is still to be understood about the subject, there are many opportunities to make contributions to applications of quantum phenomena. Some of the excitement surrounding the engineering of quantum systems is that it is so distinct from natural biology.

Differences between natural biological and man-made systems

There are significant differences between natural biology and man-made systems. Living species that occur on Earth have evolved over a long period of time in a highly energy-constrained environment. Remarkable, energy efficient, carbon-based living systems that require large biological macromolecules[4] built from smaller organic molecules exist. The combination of macromolecules and energy constraints has resulted in biology that is largely understood in terms of classical models operating under near equilibrium conditions. There are no significant quantum phenomena that dominate what is often complex system behavior. And, our everyday experiences and perception can be explained by classical physical models. The same cannot be said of complex man-made integrated electronic systems that are often created in extreme non-equilibrium conditions, operate far from equilibrium, and can rely on quantum effects for their operation.

After billions of years of evolution, biology could not and did not create the transistor. The fact is that humans created machines to do so. It is not that biology is *lazy*, rather energy constraints have excluded it from creating certain types of systems.

Transistors, fabricated with nanoscale precision using non-equilibrium processes, are routinely configured to perform calculations with an accuracy that is unachievable in biological systems. The existence of this precision in the cyber world is only possible

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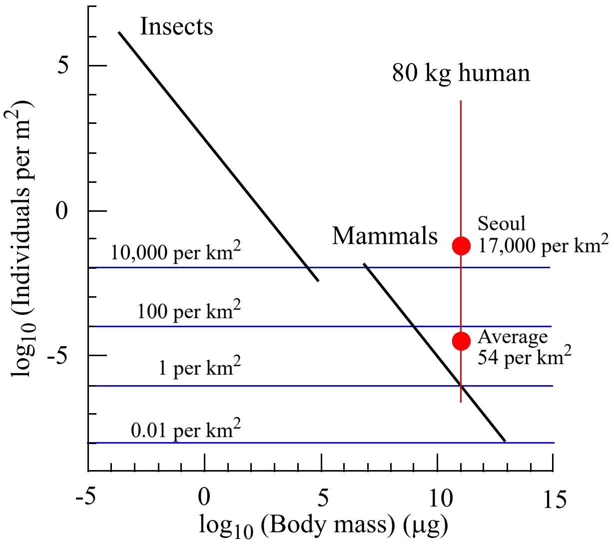
[4] The four major classes of biological macromolecules are proteins, carbohydrates, nucleic acids, and lipids.

because the transistor and other devices such as the laser diode have been created. The ability to calculate and interface to the real world are *resources* that enable novel approaches to problem solving and extending knowledge.

In contrast to evolved biological systems, *nature* contains many examples of extreme precision. For example, the existence of *identical* indistinguishable particles and apparently constant physical values such as $\hbar$. A dimensionless quantity that determines the strength of electromagnetic interaction between elementary charged particles is the fine structure constant, $\alpha = e^2/4\pi\varepsilon_0\hbar c$, which can be measured to a relative standard uncertainty of a fraction of a part per billion and does not seem to have changed its value for billions of years.

Living on the technological edge

That technology and innovation is essential for human survival is apparent when considering the sustainability of the world population. It is thought that the natural population density of mammals and insects is inversely related to the species average mass. [5] The following figure illustrates this for Earth with a land area of 148,300,000 km^2. Given an average human weight of 80 kg and assuming all land area is habitable, this suggests a maximum *natural* equilibrium trend-line population density of 1 person per km^2 corresponding to a total global population of less than 150 million. However, the world population in 2022 is near 8 billion which is 54 times greater than the maximum natural trend line value. On average there are 54 people per km^2 or one person per 1.85 hectare (4.6 acres).



The clustering of human population locally amplifies the off trend-line effect. For example, Seoul, South Korea, has a population density which is at least 17,000 per km^2 and so 17,000 times greater than the natural trend-line population density.

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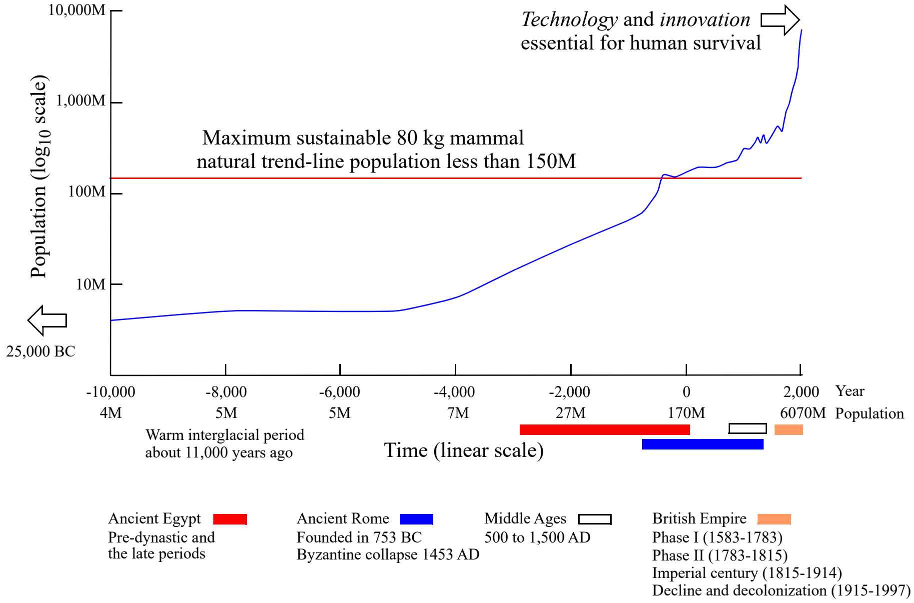
[5] John Damuth, *Nature* **290**, 699 (1981) and *Bio. J. Linnean Soc.* **31**, 193 (1987), Marina Silva and John A. Downing, *American Naturalist* **145**, 704 (1995), Pablo A. Marquet, *Science*, **289**, 1487 (2000)

World population has grown over time as science and technology developed predictive models that have subsequently been used to control processes. Most significant is the control and use of energy that enabled the industrial revolution. In more recent times efficiencies and productivity gains in services, manufacturing, resource, and supply chain management, in part due to innovation in electronic systems, has enabled continued population growth and increase in average living standard.

The following figure shows estimated world population as a function of time. [6] Notice the vertical scale is logarithmic, indicating *greater than exponential* increase in population over the past few hundred years.

8 B estimated world population (2022) is 54 times greater than trend line



Many orders-of-magnitude improvement in energy efficiency remain to be realized in electronics-based systems. However, it is unclear if this will be used to support a further increase in population or to increase average living standard.

What is apparent is that by certain metrics man-made electronic systems outperform biological systems. For example, consider honeypot ants. They can measure from 6 mm to 12 mm in length and have a brain that consists of about 250,000 neurons. [7] Even though ants don't appear to have emotions they do demonstrate intelligent behavior including the ability to communicate, search for food, display courtship, avoid their enemies, and navigate over relatively long distances. While a CMOS electronic circuit is typically not self-powered, not mobile, not autonomous, and not able to reproduce, it can contain a large number of transistors that may be used to perform precision calculations that are unattainable by any biological system. This divergence in capability between biological and man-made technologies is illustrated by the fact that there may be as many as 0.5 billion transistors per square millimeter in CMOS circuitry imple-

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6. For example, see https://en.wikipedia.org/wiki/World_population
7. The human brain is believed to contain over 86 billion brain cells.

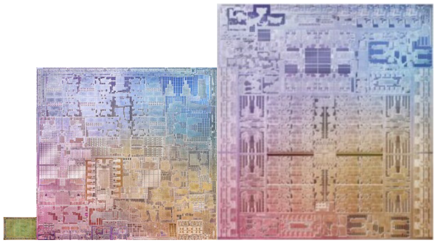
mented in a 1 nm technology. Such a density of transistors is 2,000 times greater than the density of neurons in the brain of an ant.

Scale 1 mm^2 ☐ Divergence of technology: as many as 0.5B transistors per mm^2 in 1 nm CMOS technology; 2,000 times the number of neurons in the brain of an ant.

To abstraction and beyond!

Humans, while limited by biology to classical sensory perception, seek to express themselves in different ways. This drive to share concepts and ideas, often realized as abstractions, goes back many thousands of years and is documented by, for example, the remarkable cave paintings of Pech Merle dating from before 25,000 BC. [8] Other forms of expression that have had enormous cultural impact include, speech, literature, mathematics, and music. In more recent times a pinnacle of human achievement is precision manufacturing of electrical circuits with control at the nanometer scale. This, a direct result of curiosity-driven science and application-inspired engineering, has enabled contemporary electronics, circuits that contain billions of transistors, computing, and access to opportunities in a truly extraordinary abstraction environment – the cyber world. As expected with new technologies, it may be used to advantage for the benefit of humans. However, only time will tell how society chooses to engage with the cyber world – an amazing new frontier that is a direct product of human endeavor and ingenuity.

Pech Merle



M2 processor

It seems that the contribution of computer-based systems to human creativity and development of knowledge is still in its infancy. There is much more that can and

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[8] https://en.wikipedia.org/wiki/Pech_Merle

should be done to advance the underlying technologies and to understand applications capable of changing the human experience.