Coherent control of non-Markovian photonresonator dynamics

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"Coherent control of photons"

- Coherent
 - Keep track of amplitude, phase, and maintain precise timing within a characteristic length or time scale
- Control
 - Methods to modify the behavior of a system with one or more inputs
- Photon
 - A wavelike particle of energy that produces a click on a detector
- ... and, there may be additional opportunities with *quantum* aspects of photons
 - Quantum
 - No accepted measure of "quantumness"
 - Classical analogies often exist (e.g. the coherent oscillator state) except for:
 - Wave-particle duality
 - Identical indistinguishable particles
 - Linearity
 - Entanglement

Motivation: Demonstrate a path to *intuitive* control of transient photon dynamics (in quantum systems)



Modified from Rabitz et al., 2010: General unifying features of controlled quantum phenomena: https://arxiv.org/pdf/1003.3506

The single photon wave function

- Single photons in a sourceless medium obey the equation $i\hbar\partial_t\psi_\sigma(\mathbf{r},t) = \hbar c\sigma \nabla \times \psi_\sigma$ where σ is the helicity of the photon.
- If $\psi_{\sigma} = \left(\frac{\epsilon_0}{2}\right)^{1/2} (E + i\sigma cB)$, then the above equation yields Ampere's and Faraday's Laws, i.e. Maxwell's equations (with the additional condition that the field is divergenceless).
- In other words, the real and imaginary parts of the photon wave function obey Maxwell's equations.
- Thus all methods used for solving classical electromagnetic problems can be used for the *single* photon (e.g. matrix propagation method).

The single photon wave function

- It is (*now*) believed that a single photon wave function may be used to describe photon *energy density* $U(x,t) = |\Psi(x,t)|^2$
 - Studied theoretically, including
 - I. Bialynicki-Birula, Acta Phys. Pol. 86, 97 (1994)
 - B. J. Smith and M. G. Raymer, New Journal of Physics **9**, 414 (2007)
- Unitary dynamics of photon wave function propagating in x-direction in lossless dielectric media may be modeled as phase-coherent sum of linearly polarized planewave basis functions, each of amplitude a_n and oscillating at frequency ω_n

$$\Psi(x,t) = \sum_{n} a_{n} \psi_{n}(x) e^{-i\omega_{n}t}$$

• The lossless dielectric material may be characterized by μ_r and ε_r , and ψ_n satisfies $\nabla \times ((\mu_0 \mu_r)^{-1} \nabla \times \psi_n(x)) - \omega_n^2 \varepsilon_0 \varepsilon_r \psi_n(x) = 0$ with boundary conditions between region 1 and 2 at position x_0 such that

$$\psi_n\big|_{x=x_0-\delta}=\psi_n\big|_{x=x_0+\delta}$$

 $\frac{1}{\mu_{r1}} \frac{\partial \psi_n}{\partial x} \bigg|_{x=x_0-\delta} = \frac{1}{\mu_{r2}} \frac{\partial \psi_n}{\partial x} \bigg|_{x=x_0+\delta}$

and refractive index $n_{\rm r} = \sqrt{\mu_{\rm r}} \sqrt{\varepsilon_{\rm r}}$

- Solve in space and time and assume photon coherence time, $\tau_{\rm Coh}$, greater than any other characteristic time

The Fabry-Perot optical resonator

- Example dielectric structure: Two quarter-wave dielectric mirrors with $n_r=2.5$ spaced $L_c=2\lambda_0$ apart creates Fabry-Perot optical *resonator* that is *coupled to the continuum*
- Resonant wavelength λ_0 =1500 nm, ω_0 =2 π/τ_0 =2 π x200 THz, τ_0 =5 fs
- Resonant photon energy E_0 =0.826 eV
- $Q=\omega_0/\gamma$, Lorentzian spectral FWHM= \hbar/τ_Q , $\gamma=1/\tau_Q$
- Classical time-domain response is e^{-t/τ_Q} and τ_Q is resonant state lifetime
- Resonator round-trip time defined as $\tau_{\rm RT}=2\pi/\Delta\omega=1/\Delta f$ where Δf is the frequency spacing between adjacent spectral transmission peaks



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Reflectivity of single quarter-wave dielectric mirror

- Example dielectric structure: Single quarter-wave dielectric mirror with $n_r=2.5$
- Resonant wavelength λ_0 =1500 nm, ω_0 =2 π/τ_0 =2 π x200 THz, τ_0 =5 fs
- Resonant photon energy $E_0=0.826 \text{ eV}$
- Transmission is a *slowly* varying function of photon energy, ^{1.2}

$$t^{2} = \frac{1}{1 + \left(\frac{k_{1}^{2} - k_{2}^{2}}{2k_{1}k_{2}}\right)^{2} \sin^{2}(k_{2}L)}$$

On resonance $k_{1} = \frac{2\pi}{\lambda_{0}}$, $k_{2} = \frac{2\pi n_{r}}{\lambda_{0}}$, $L = \frac{\lambda_{0}}{4n_{r}}$, so that

$$\frac{1}{1 + \left(\frac{1 - n_{\rm r}^2}{2n_{\rm r}}\right)^2}$$

- System requires $t^2 + r^2 = 1$
- $t^2 = r^2 = 1/2$ when $n_r = 1 + \sqrt{2}$ For field $t = \sqrt{t^2}$ and $r = \sqrt{r^2}$
- *t* and *t*² less dependent on photon energy as:

$$m_{\rm r} \rightarrow 1, t^2 \rightarrow 1$$

 $m_{\rm r} \rightarrow \infty, t^2 \rightarrow 0$

 π phase shift on reflection from semi-infinite slab, ψ_{r}



Transient response of single-photon (or classical E&M) pulse incident on Fabry-Perot cavity at resonance

- Rectangular photon pulse with center frequency that is *on resonance at* wavelength λ_0 =1500 nm, ω_0 =2 π/τ_0 =2 π x200 THz, τ_0 =5fs
- Quarter-wave $(\lambda_0/4n_r)$ dielectric mirror with $n_r=2.5$, cavity length $L_c=15\lambda_0$, cavity round-trip time $\tau_{\rm RT}=2\pi/\Delta\omega=30\tau_0=150$ fs
- Q=193, τ_Q=Q/ω₀=153fs

 $\tau_0 = 5 \text{fs}$

 $\tau_{\rm p}$ =1432fs $\tau_{\rm RT}$ =150fs

 $\tau_{\rm Q}$ =153fs

Optical resonator: λ_0 =1500nm, E_0=0.827eV, n_r=2.5, L_C=15\lambda_0, E_{spread}=0.207eV, L_0=286.5\lambda_0 20 Rectangular photon pulse of length $L_{\rm P}$ =286.5 $\lambda_{\rm O}$ ($\tau_{\rm p}$ =1432fs) and center frequency f_0 = 200 THz 15 $|\psi(x,t)|^2$ (τ_0 =5fs) propagating from left to right 10 5 0 200 1000 400 600 800 1200 Position, x (um) 3 Refractive index, n_r Fabry-Perot dielectric resonator, $L_{\rm C}$ =15 λ_0 , $\tau_{\rm RT}$ =150fs, $\tau_{\rm O}$ =153fs 0 200 400 600 800 1000 1200 Position, x(um)

Transient response of single-photon pulse incident on Fabry-Perot cavity at resonance

• When photon round-trip time in resonator τ_{RT} is comparable to envelope response time τ_{Q} it is possible to probe the internal (ring-down) structure of the resonator



Transient response of single-photon pulse incident on Fabry-Perot cavity at resonance



Transient response of single-photon pulse incident on Fabry-Perot cavity at resonance

- Multiple cavity round-trip times required to build-up steady-state behavior
- Rectangular photon pulse with center frequency that is on resonance has characteristic transient reflection at leading and trailing edge
 - Reflection depends on frequency components contributing to pulse shape
 - Reflection always greater than zero for pulse

Optical resonator: λ_0 =1500nm, E $_0$ =0.827eV, n = 2.5, L $_C$ =15 λ_0 , T $_0\omega_0$ =900, E $_{spread}$ =0.207eV

Burst of reflected light from trailing edge of pulse

Burst of reflected light from leading edge of pulse

No reflected light at resonant wavelength



Transient response of single-photon pulse incident on Fabry-Perot cavity at resonance

• Calculated transmitted single-photon energy density, $|\psi(x_R, t)|^2$ at $x_R = 742.8 \mu m$



Experimental validation using Fabry-Perot resonator in waveguide

- Because equations governing single-photon wave function evolution are similar to the Helmholtz equation, experiments using classical electromagnetic resonators can validate qualitative behavior.
- Example:
 - Resonant frequency, f_0 =15.234 GHz (E_0 =63 µeV)
 - Measured permittivity of teflon, ε_r =2.050
 - Measured loss tangent, δ =0.0005







Experimental validation using Fabry-Perot resonator in waveguide

- Measured transmitted electromagnetic energy density in time-domain U(t) ($|mV|^2$ into 50 Ω)
- Resonant frequency, f_0 =8.0620 GHz (E_0 =33 µeV)
 - 1/25,000 scale reduction in frequency from optical to RF
- 3 quarter-wave pair DBR in teflon
- Round-trip time in resonator $\tau_{\rm RT}$ =12 ns
- Resonator Q=582 corresponds to $\tau_{\rm Q}$ =11.5 ns (red curve)



• Long pulse time $\tau_p = 230 \text{ ns} \gg \tau_{RT}$, τ_Q



Uncontrolled short single-photon pulse in cavity ring-down



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resonances is $\Delta \omega = \pi c / L_c n_r$

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Uncontrolled single-photon cavity ring-down



Experimental validation using Fabry-Perot resonator in waveguide

- Can also probe the internal structure of the Fabry-Perot resonator using *short* electromagnetic pulse time τ_p =7 ns < $\tau_{\rm RT}$, $\tau_{\rm Q}$
- Round-trip time in resonator $\tau_{\rm RT}$ =13.7 ns
- Resonator Q=633 corresponds to $\tau_{\rm Q}$ =12.5 ns (red curve)
- RF switch rise time is 2 ns
- Measured transmitted electromagnetic signal in time-domain $|\rm mV|$ into 50 Ω



Coherent control of *transient dynamics*

- Zero-energy ground-state is a *guaranteed* control point
- Question: How do you *stop* a bell ringing ?
 - The "ringing bell" could be *excitations* of a molecule, a crystal, a device, ...



Coherent control of *transient dynamics*

- Question: How do you *stop* a bell ringing ?
- Answer: You hit it ! (... in a *very* controlled and precise way)



Control field generator

System

Controlled single-photon zero cavity ring-down



pulse energy

density

Space

 Formal control methods replaced by *intuitive* raytracing of *resonant* part of photon field

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Controlled single-photon zero cavity ring-down



Transmitted single-photon pulse energy density (cancellation of ring-down)

 $au_0 = 5 \text{ fs}$ $au_p = 80 \text{ fs}$ $au_{RT} = 150 \text{ fs}$ $au_Q = 153 \text{ fs}$

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Resonator energy density output as function of control pulse energy density



Coherent control of single-photon resonator output pulse using control pulse

- Better than 1:10⁴ cancellation using simple control pulse protocol
- Cancellation of residue requires better control pulse match to Fabry-Perot transfer function or slower (smaller bandwidth) operation



Experimental validation using Fabry-Perot resonator in waveguide



 f_0 = 8 GHz, $\tau_{\rm Coh}$ = 1 s, τ_0 = 125 ps, $\tau_{\rm p}$ = 7 ns, $\tau_{\rm RT}$ = 13.7 ns, $\tau_{\rm Q}$ = 12.5 ns

- Coherent control of resonator *short* output pulse using single *short* control pulse
- Short electromagnetic pulse of width $\tau_{\rm p}$ =10 ns < $\tau_{\rm RT}$, $\tau_{\rm Q}$
- Round-trip time in resonator $\tau_{\rm RT}$ =13.7 ns
- Resonator Q=633 corresponds to $\tau_{\rm Q}$ =12.5 ns (red curve)
- Measured transmitted electromagnetic signal in time-domain |mV| into 50 Ω







Coherent control of single-photon resonator output pulse using one backward control pulse



Coherent control of single-photon resonator output pulse using one backward control pulse



Transmitted photon energy density as single pulse

 $\tau_0 = 5 \text{ fs}$ $\tau_p = 80 \text{ fs}$ $\tau_{\text{RT}} = 150 \text{ fs}$ $\tau_{\text{Q}} = 153 \text{ fs}$

Backward propagating control pulse

Coherent control of single-photon resonator output pulse using two backward control pulses



Coherent control of single-photon resonator output pulse using two backward control pulses



Coherent control using one forward and one backward control pulse (or a very small pulse can control two large pulses)



Incident photon

pulse energy Space

density

- Two pulses reflected with *no* ring-down
- Note small energy in backward propagating control pulse

Coherent control using one forward and one backward control pulse



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Coherent control using one forward and two backward control pulses



Incident photon

pulse energy Space

density

- propagating control pulses incident on resonator
 Single pulse transmitted and single pulse reflected
- Single pulse transmitted and single pulse reflected with *no* ring-down

Coherent control using one forward and two backward control pulses



Coherent control of single-photon resonator output pulses using *multiple* control pulses

- Lead pulse and three control pulses incident on resonator
- Three identical pulses transmitted and dual pulse reflected with *no* ring-down



Coherent control of single-photon resonator output pulse using *three* control pulses





Geometric series using coherent control pulses to *confine* photon energy density in resonator



|eⁱ∕r| >1.

Coherent control



Markovianity measure D(t) of single photon

- System (defined as some region of domain) coupled to continuum at $\pm\infty$
- Unitary evolution of initial state eventually dissipates
- Define spatial region A in domain and consider freely propagating photon pulse through this region
- Under these conditions one may expect any measure of Markovianity in region A to indicate Markovian behavior
 - Information leaks out of region A as the photon energy density decays into the continuum



Hilbert-Schmidt measure D(t) and Markovianity

 Two initially non-interacting (non-overlapping) photon pulses with unitary evolution and initial states such that $\psi_2(x, t) = \psi_1(x, t + \tau_M)$ for fixed delay τ_M have Hilbert-Schmidt measure in spatial region A given by (Lorenzo Campos Venuti)

$$D(t) = \frac{\sqrt{\left(\int_{A} |\psi_{1}(x,t)|^{2} dx\right)^{2} + \left(\int_{A} |\psi_{2}(x,t)|^{2} dx\right)^{2} - 2\left|\int_{A} \psi_{1}^{*}(x,t)\psi_{2}(x,t)dx\right|^{2}}{\sqrt{2}}$$

- The system is considered Markovian if initial states $\psi_1(x, t)$ and $\psi_2(x, t)$ are both in spatial region A and D(t) subsequently decreases monotonically with time
- Example: non-interacting rectangular photon pulse propagating in free-space



Hilbert-Schmidt measure D(t) of photon interacting with Fabry-Perot dielectric resonator

- A (blue curve) left of dielectric resonator with some energy density flow into region
- B (black curve) which is in dielectric resonator with energy density flow into region
- *C* (red curve) to right of dielectric resonator
- *Note*: Extent of region in domain and definition of system, subsystem, bath, etc., is arbitrary



Hilbert-Schmidt measure D(t) of photon interacting with Fabry-Perot dielectric resonator

- Incident pulse-width $\tau_{\rm p}$ < $\tau_{\rm RT}$, $\tau_{\rm Q}$
- Single incident pulse produces multiple output pulses (ring-down) spaced at cavity round-trip time and of decreasing energy density as resonator decay e^{-t/τ_Q}



Hilbert-Schmidt measure D(t) of photon interacting with Fabry-Perot dielectric resonator

- A (blue curve) left of dielectric resonator with some energy density flow into region
- *B* (black curve) which is in dielectric resonator with energy density flow into region
- *C* (red curve) to right of dielectric resonator



Integrated non-Markovianity, $DI(t) = \sum_{i} (\Delta D(t_i)|_{\text{positive}})$



With single control pulse at $\tau = \tau_{\rm BT}$ Optical resonator: λ_n =1500nm, E_0=0.827eV, n_r=2.5, L_C=15 λ_0 , E_{spread}=0.207eV, τ_M =0.225ps Mamp0525T030M45.tif Non-Markovian 0.8 0.6 ⁾ ص 0.4 0.2 0.2 0.4 0.6 0.8 1.2 1.4 1.6 1.8 0 1 Time, t (ps) Optical resonator: λ_{D} =1500nm, E_{D} =0.827eV, n_{r} =2.5, L_{C} =15 λ_{D} , E_{spread} =0.207eV, τ_{M} =0.225ps Mamp0525T030M45DI.tif 0.9 Non-Markovian $\tau_0 = 5 \text{fs}$ 0.2 $\tau_{\rm RT}$ =150fs 0.1 $\tau_{\rm Q}$ =153fs $\tau_{\rm M}$ =225fs 0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8

Time, t (ps)

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Integrated non-Markovianity, $DI(t) = \sum_{i} (\Delta D(t_i)|_{\text{positive}})$



Integrated non-Markovianity, $DI(t) = \sum_{i} (\Delta D(t_i)|_{\text{positive}})$

No control pulse Cavity length L=120 x λ_0 Sub-space B' is left-side half-cavity of length L/2



Summary: Coherent control of single-photon energy density in a resonator

- Photonic resonator with lossless dielectric mirrors driven by phase-coherent source is open system coupled to continuum that evolves with unitary dynamics
 - Photon wave function, $\Psi(x,t)$, describes single-photon energy density, $U(x,t) = |\Psi(x,t)|^2$
 - Multiple resonator round-trip times, $\tau_{\rm RT}$, required to build-up steady-state behavior
 - Steady-state behavior evolves exponentially during characteristic resonator time, $\tau_{\rm Q}$
- *Transient behavior* controlled by incident waveform
 - Non-Markovian dynamics because mirror reflections and energy stored in resonator
 - Can eliminate all energy density in resonator in less than one round-trip time, $\tau_{\rm RT}$
 - Can control exact number of identical transmitted and reflected pulses at multiples of round-trip time, $\tau_{\rm RT}$
 - Can use control to pass long pulses, $\tau_{\rm P} > \tau_{\rm RT}$, through resonator
 - Control of transient behavior is also control of Markovianity (and hence information flow)
 - Non-Markovianity may be viewed as resource for quantum information processing
- Natural time scales are { τ_0 , $\tau_{\rm RT}$, $\tau_{\rm Q}$ } < $\tau_{\rm Coh}$
 - Resource for manipulation of single-photon quantum states
 - Use waveform as sensor probe of resonant structures (inverse problem)

Coherent control of single-photon transient dynamics in a Fabry-Perot resonator

A.F.J. Levi, L. Campos Venuti, T. Albash, and S. Haas, "Coherent control of non-Markovian photon resonator dynamics" Phys. Rev. A (2014)

OBJECTIVE:

- Demonstrate coherent quantum control of single-photon dynamics in an optical storage device
- Apply techniques developed to demonstrate control of non-Markovianity

APPROACH

Use photon pulse injected into Fabry-Perot resonator as model system coupled to continuum and simulate in space-time domain
 Exert precise control on photon dynamics using coherent control pulses and intuitive resonant control protocol
 Use L. Campos Venuti's computationally-efficient measure of non-Markovianity: N. Chancellor, C. Petri, L. Campos Venuti, A.F.J. Levi, and S. Haas, Phys. Rev. A (2014)





ACCOMPLISHMENTS: Successfully demonstrated control of singlephoton transient dynamics in Fabry-Perot resonator and measure of non-Markovianty

 A first step to exploitation of non-Markovian transient photon dynamics as a resource in coherent quantum systems
 Established methodology and techniques for further study



Coherent control of single-photon transient dynamics for logic

A. Abouzaid, F. Wang, S. Gupta, and A.F.J. Levi

OBJECTIVE:

 Demonstrate coherent quantum control of single-photon dynamics may be applied to boolean logic and perform exhaustive search in minimal linear device design sub-space for all feasible logic operations

APPROACH

> Basic (minimal) device building block is symmetric 50:50 singlephoton beam-splitter

➢ Formally enumerate all tree structures up to depth of k in which tree-structures have no feedback and no re-convergent fanout

> Combine two phase shifters/Modulators with one beam splitter to reduce the enumeration complexity

Use physical model to validate results

ACCOMPLISHMENTS: Exact suppression of reflection at beamsplitter using control pulse

 Optical pulse contains broad spectrum of frequency components that interact with symmetric 50:50 singlephoton beam-splitter
 Optimal control pulse to suppress reflection requires search for coherent single-photon phase and amplitude field parameters



ACCOMPLISHMENTS: Enumerator discovers all feasible boolean logic tree-structures

High-level simulator checks if feasible solution exists (to depth k in tree-structure) for a designated boolean function
 If no feasible solution is found by the high-level simulator, then that logic function cannot be implemented using only linear components connected in a tree-structure

➢ For any *n*-input configuration created using only linear components connected in a tree-structure, if a feasible solution exists for a given boolean function in the high level simulation, then the boolean function is implementable in the low-level simulator



ACCOMPLISHMENTS: Successfully enumerated all boolean logic using minimal components and constraints

Constrained to single-photon, beam-splitter, amplitude modulator, phase shifter, and tree-structure

 Components indicate events, i.e. an interaction between pulses and a particular component at some point in time and space
 Successfully demonstrated NAND and so complete for boolean logic

- > Also, NOT, OR, XOR, XNOR, and multiplexing
- > AND and NOR are not feasible

> Reshaping, retiming, and re-amplifying (3R) output may be achieved for classical light using a saturable absorber, however, the single-photon version of 3R is unknown

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