Optimal design using negative refractive index as a new degree of freedom: JOSA paper

- **One-dimensional electromagnetic resonator**

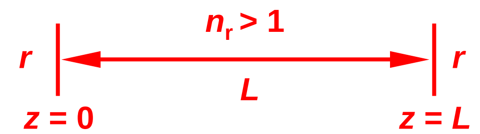
$$\nabla\times\left(\mu_0\mu_r\right)^{-1}\cdot\nabla\times\mathbf{E}-\omega_0^2\varepsilon_0\varepsilon_r\mathbf{E}=0$$

$$E_x\big|_{z=z_0-\delta}=E_x\big|_{z=z_0+\delta}$$

$$\frac{1}{\mu_1}\frac{\partial E_x}{\partial z}\bigg|_{z=z_0-\delta}=\frac{1}{\mu_2}\frac{\partial E_x}{\partial z}\bigg|_{z=z_0+\delta}$$

- **Strong correlations between measurable quantities in *conventional* DBR resonator of cavity length *L* can be discovered analytically or numerically**
  - **Electromagnetic energy in cavity *W* ~ *L***
  - **FWHM in transmission spectrum γ ~ 1/ *L***
  - **Characteristic response time τ = 1/γ ~ *L***
  - **Resonance wavelength shift Δλ ~ *L***
- **New functionality by seeking degrees of freedom that eliminate correlations**
  - **Negative refractive index** $n_r=\sqrt{\varepsilon_r}\sqrt{\mu_r}$
    - **Find configurations that decouple *W*, τ, and Δλ**
- **Will show that, compared to any given conventional device design, negative index allows:**
  - **Up to twice the electromagnetic energy density**
  - **Always a faster characteristic response time**

Conventional resonator



$$\left|\mathbf{E}(\omega)\right|^2=\frac{I_{max}}{1+\left(\frac{2\mathcal{F}}{\pi}\right)^2\sin^2\left(\frac{2\pi\omega}{\Delta\omega}\right)}$$

$$\Delta\omega=\frac{2\pi c}{2n_r L}$$ **Mode spacing**

$$\mathcal{F}=\frac{\pi\sqrt{r}}{1-r}$$ **Finesse**

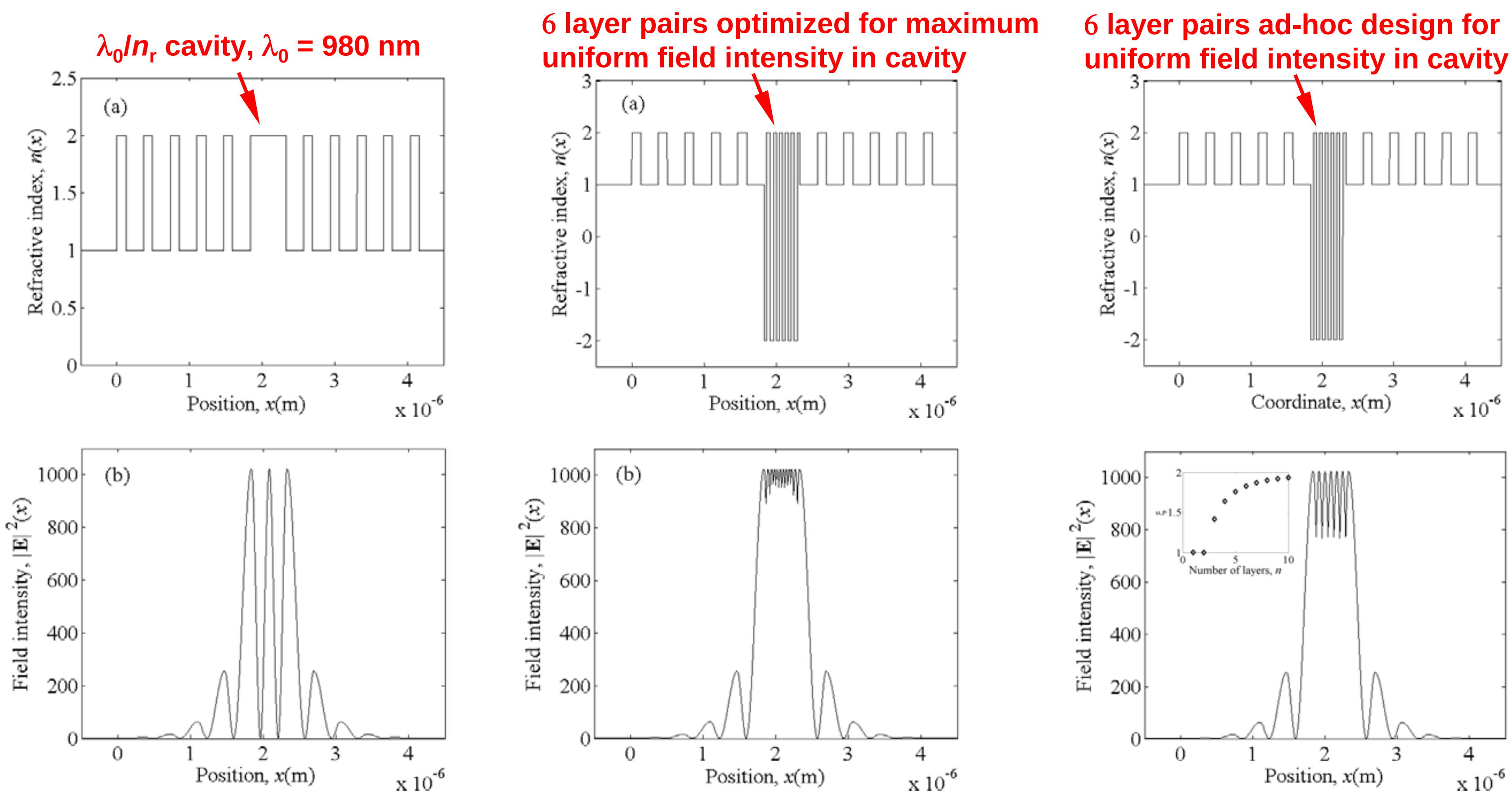
$$I_{max}=\frac{\left|\mathbf{E}_0\right|^2}{\left(1-r\right)^2}$$ **Peak intensity**

$$\gamma=\frac{1}{\tau}=\frac{\Delta\omega}{\mathcal{F}}=\frac{\pi c}{n_r L}\frac{1}{\mathcal{F}}$$

$$W\propto\int_0^L\left|\mathbf{E}_x\right|^2(z)dz$$

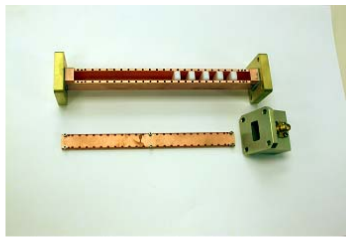
Optimal design using negative refractive index as a new degree of freedom

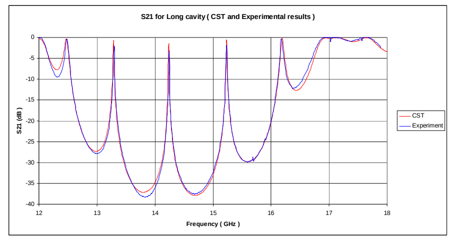
- **Negative refractive index to control electromagnetic field intensity**
  - **Maximizing electromagnetic field density in a resonant cavity**
    - **Up to 2x increase in cavity energy density, approaches uniform field distribution!**
    - ***Discovered* using optimal design**
      - **Ad-hoc studies: single dielectric pair sub-wavelength cavity resonator, N. Engheta, IEEE Ant. Wireless Prop. Lett. 1, 1536 (2002),**

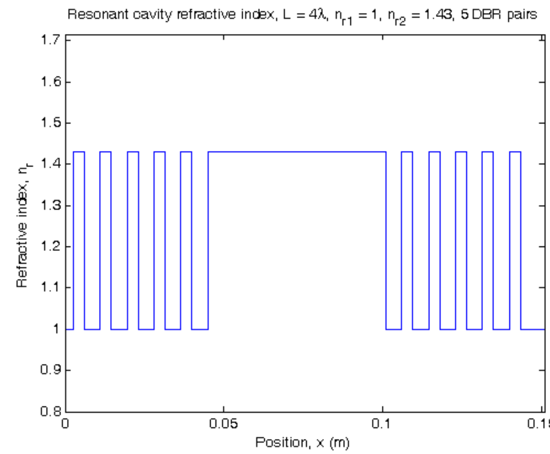


Optimal design using negative refractive index as a new degree of freedom

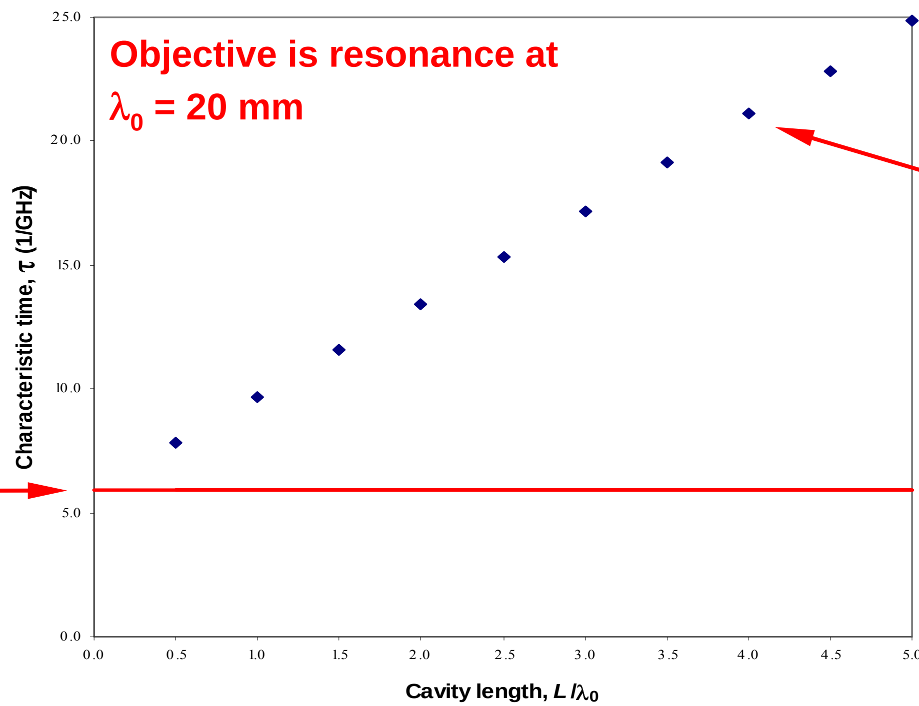
- **Example: 15 GHz (λ_0 = 20 mm) electromagnetic resonator with zero phase accumulation**
  - ***Separation* of response time τ from cavity length *L***







Value of τ at intercept $L = 0$ determined only by the reflectivity of the DBR mirrors



Conventional resonator requires *discrete* cavity length of

$L = n\lambda_0/2n_r$

where *n* is a non-zero positive integer

Zero phase accumulation resonator allows cavity to have *any* length *L*

Optimal design using negative refractive index as a new degree of freedom

- **Example: 15 GHz (λ_0 = 20 mm) electromagnetic resonator**
  - **Compared to conventional resonator, zero phase accumulation device decouples resonator transmission line width (response time τ) from cavity length *L***
    - ***Independent design variables***

Conventional resonator requires *discrete* cavity length of

$L = n\lambda_0/2n_r$

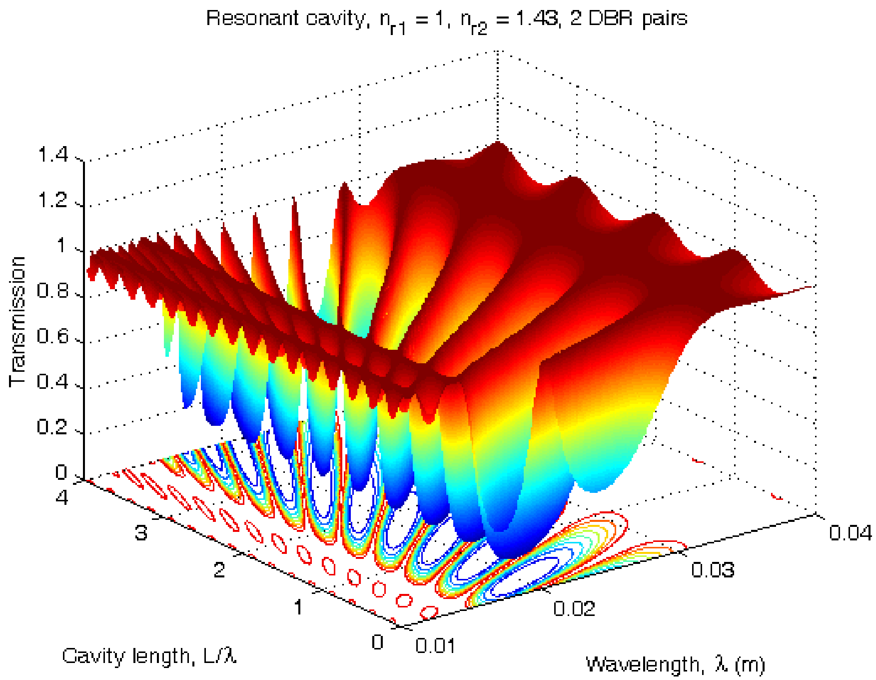
where *n* is a non-zero positive integer

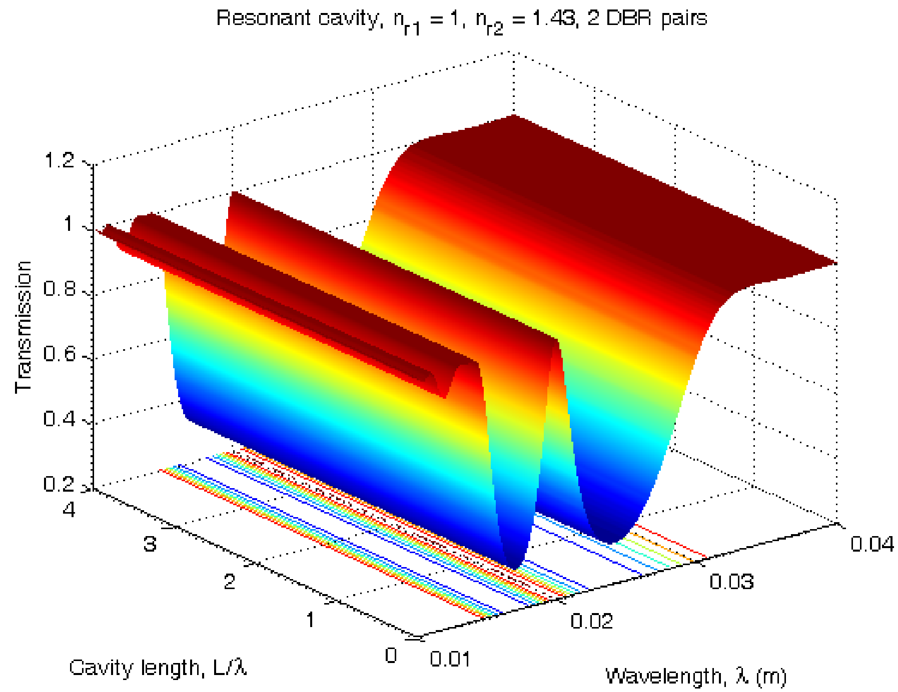
Transmission (λ, *L*) solutions complex

Zero phase accumulation resonator allows cavity to have *any* length *L*

Transmission (λ, *L*) solutions simplified

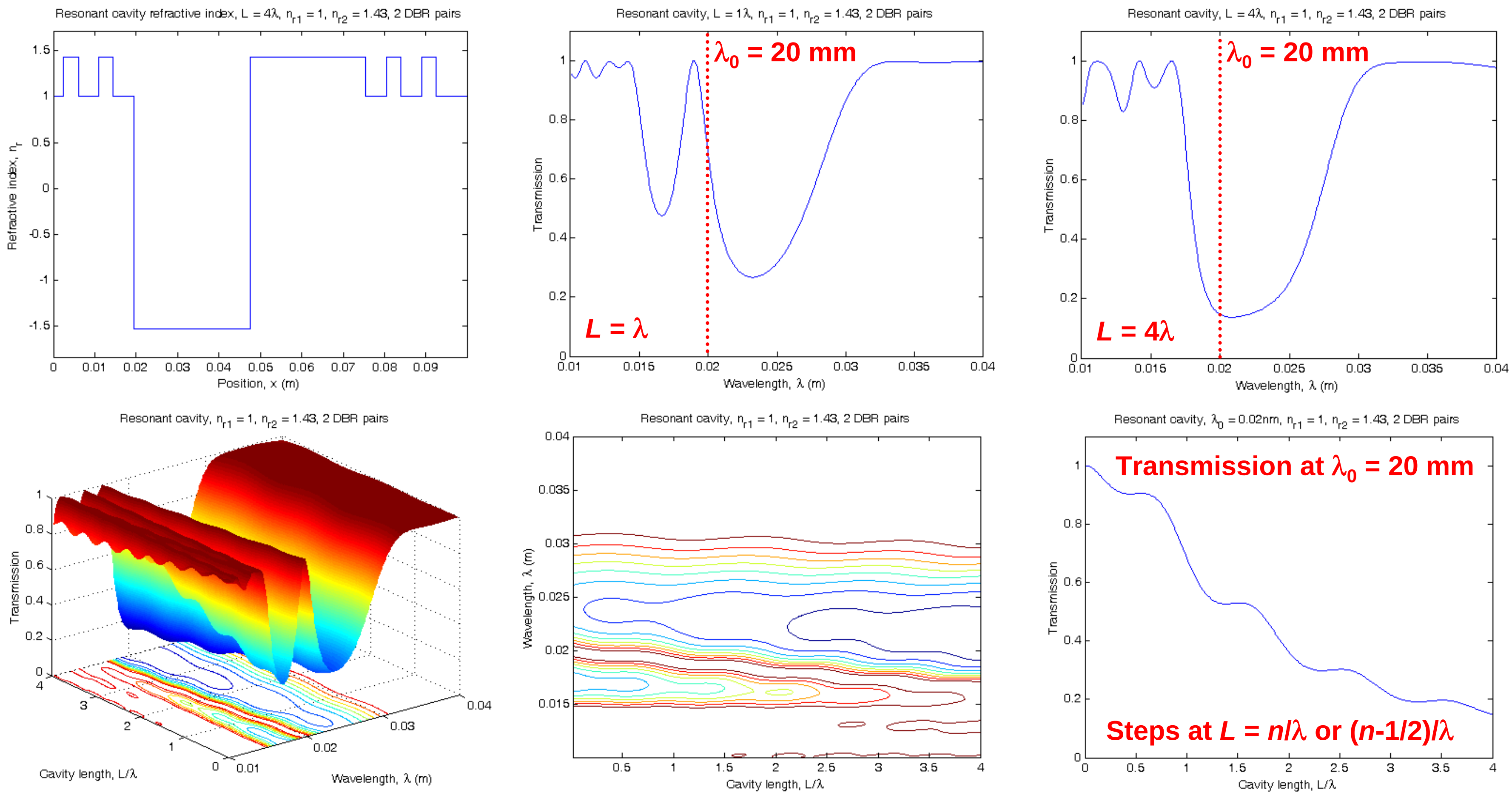
Decouple *L* from τ





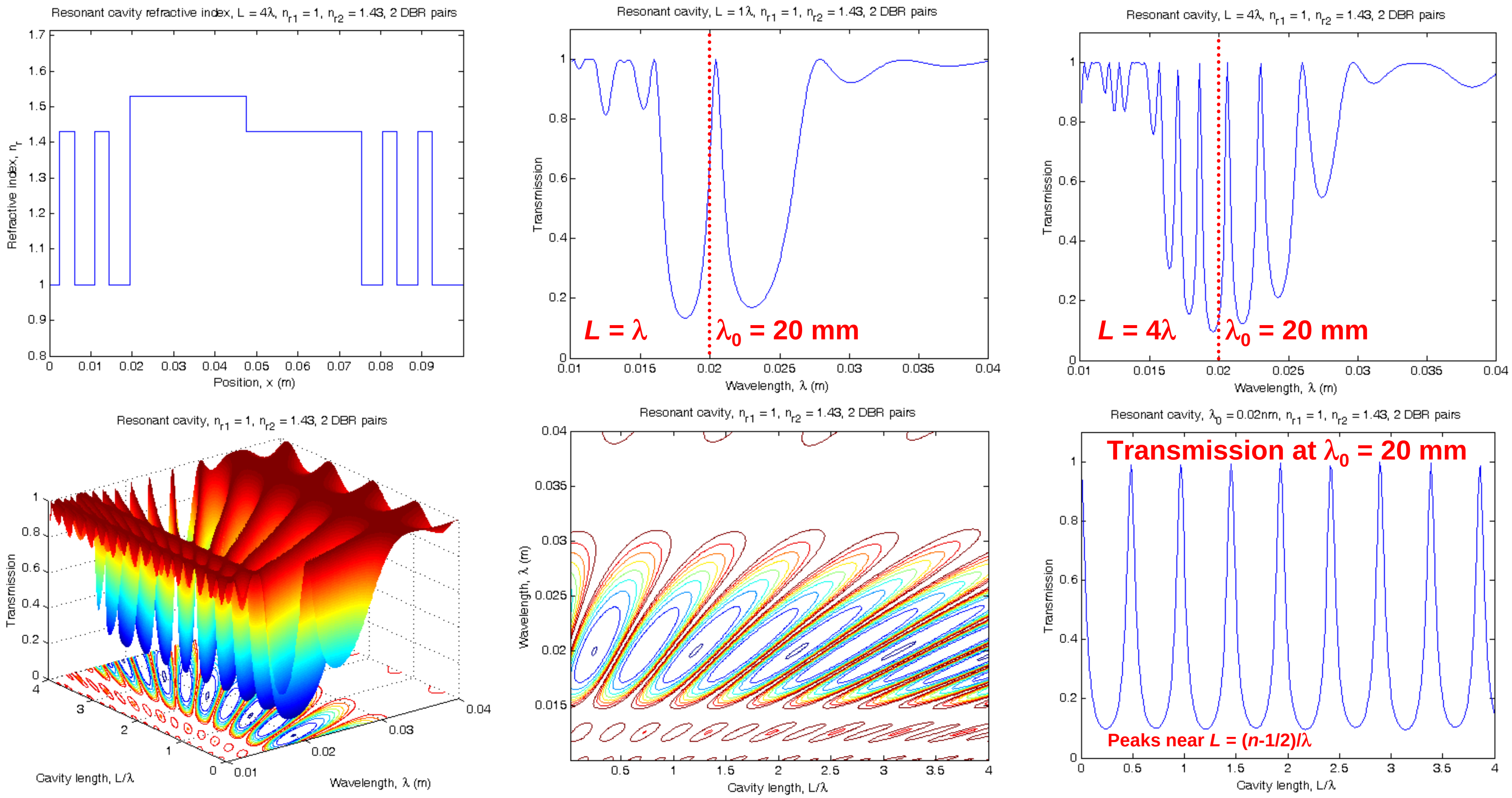
Optimal design using negative refractive index as a new degree of freedom

- Zero phase accumulation sensitivity to parameter variation
- Example: 15 GHz (λ_0 = 20 mm) electromagnetic resonator
  - 2 DBR pairs, 7% mismatch in negative and positive cavity index n_{c1} = -1.53, n_{c2} = 1.43
    - Stepped peak transmission walk-off



Optimal design using negative refractive index as a new degree of freedom

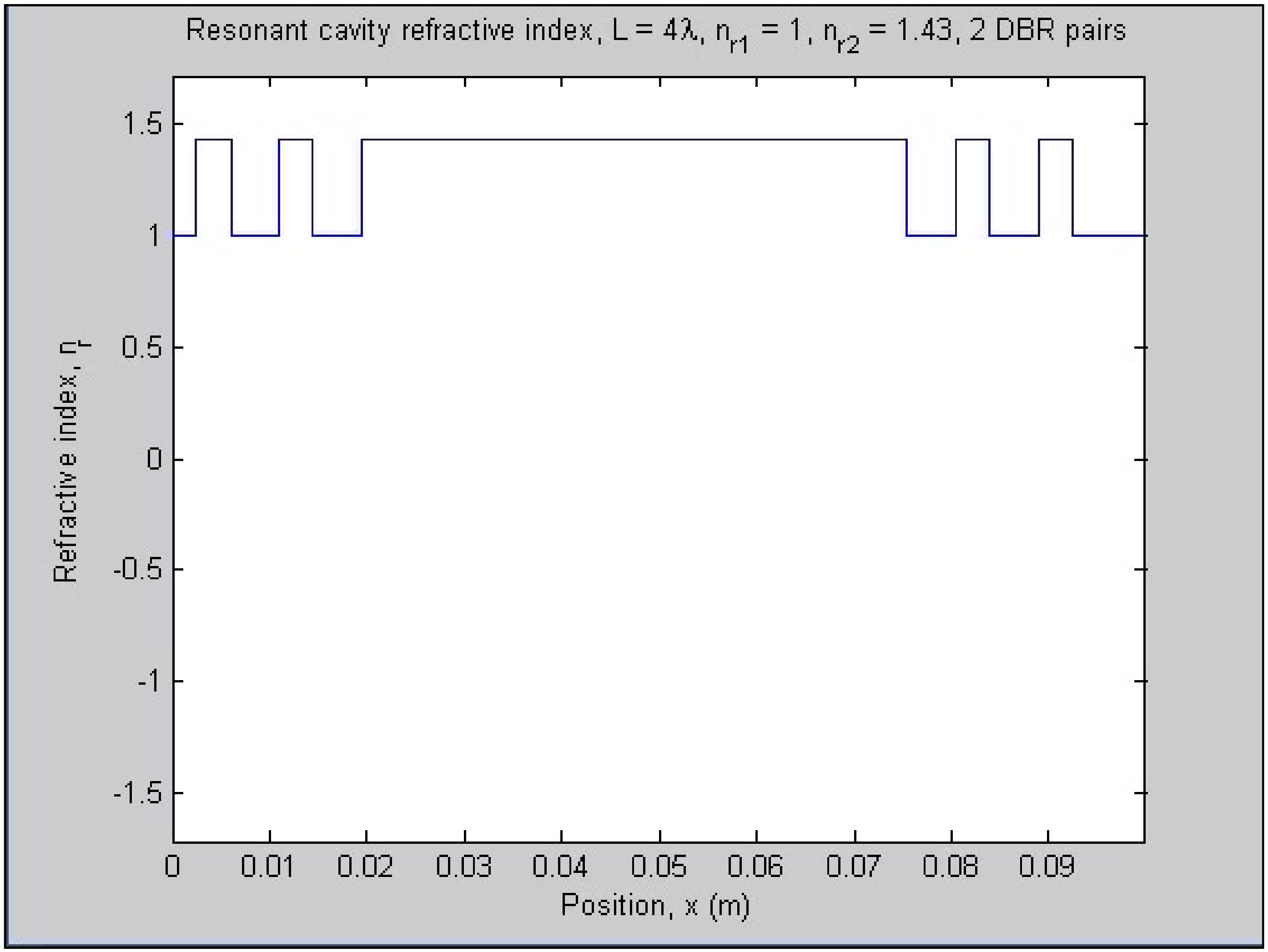
- **Conventional phase accumulation sensitivity to parameter variation**
- **Example: 15 GHz (λ_0 = 20 mm) electromagnetic resonator**
  - **2 DBR pairs, mismatch in positive cavity index n_{c1} = 1.53, n_{c2} = 1.43**
    - **Poorly behaved with *oscillations* in peak transmission**



Optimal design using negative refractive index as a new degree of freedom

- Solution space as function of fraction of negative index material in cavity
- Example: 15 GHz (λ_0 = 20 mm) electromagnetic resonator
  - 2 DBR pairs, $n_{c1} = 1.43$, $n_{c2} = -1.43$

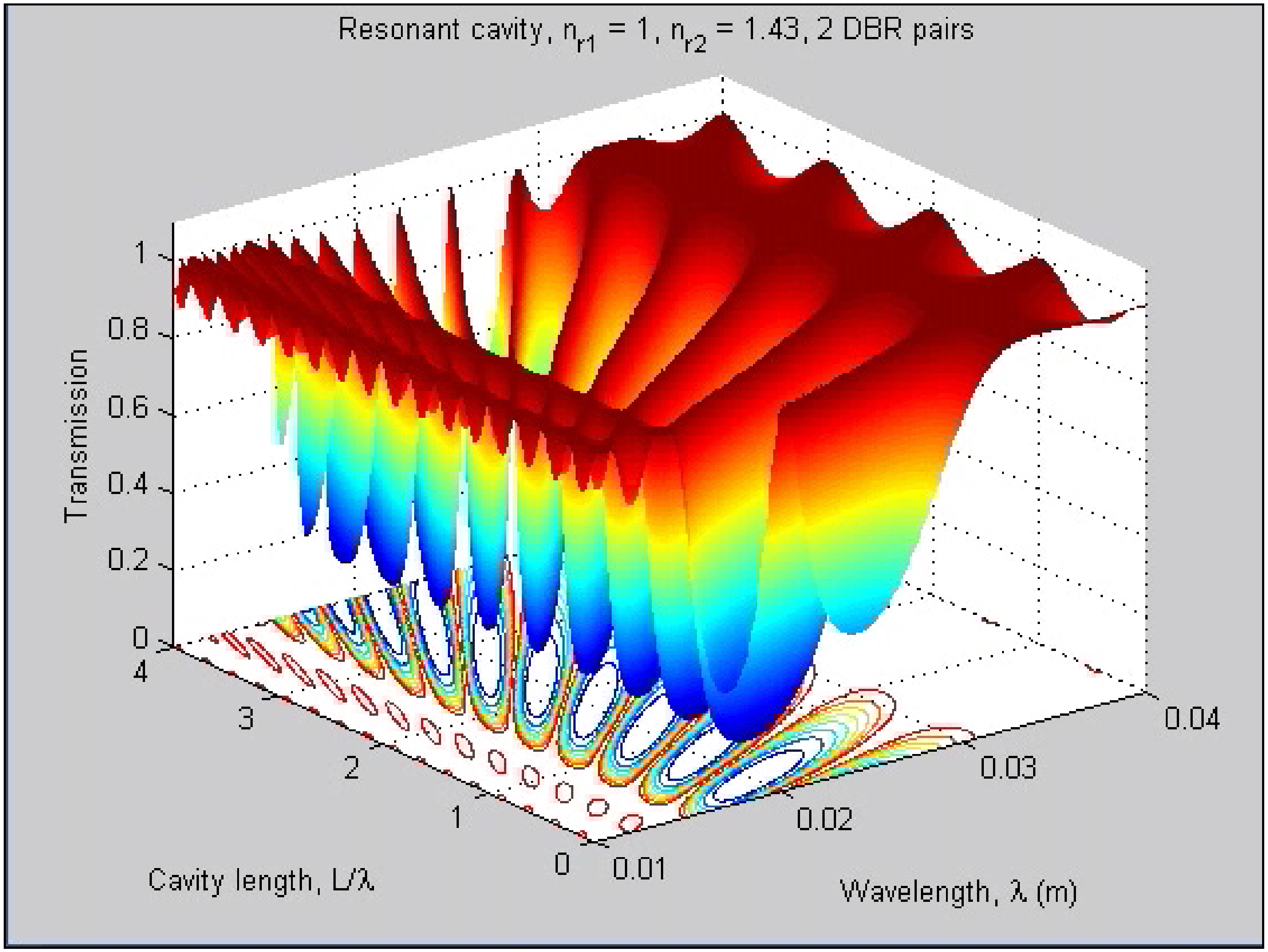
Click image to run video



Optimal design using negative refractive index as a new degree of freedom

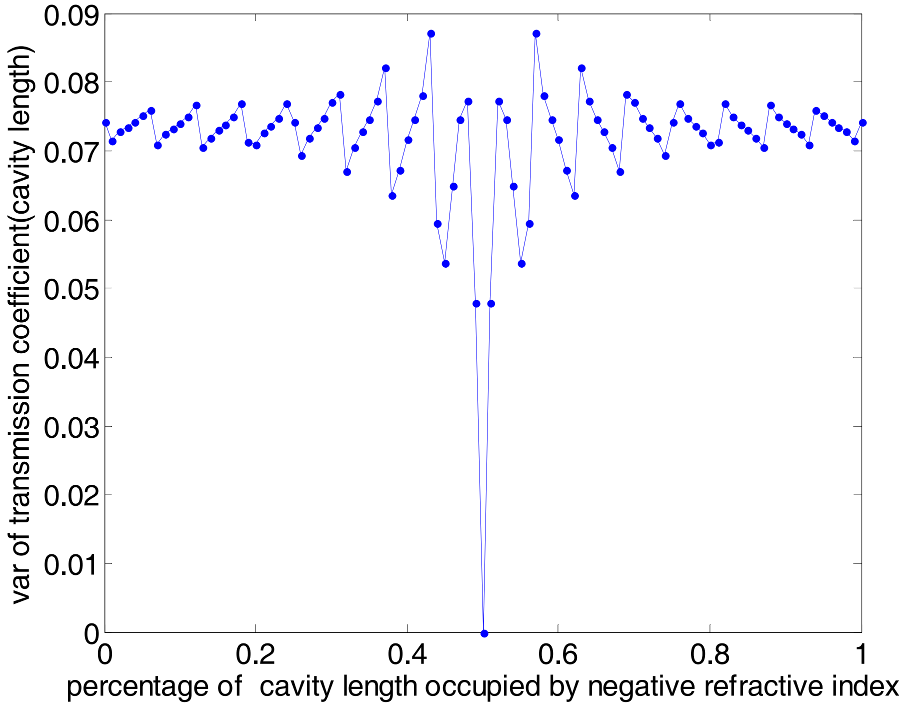
- **Solution space as function of fraction of negative index material in cavity**
- **Example: 15 GHz (λ_0 = 20 mm) electromagnetic resonator**
  - **2 DBR pairs, n_{c1} = 1.43, n_{c2} = -1.43**

Click image to run video

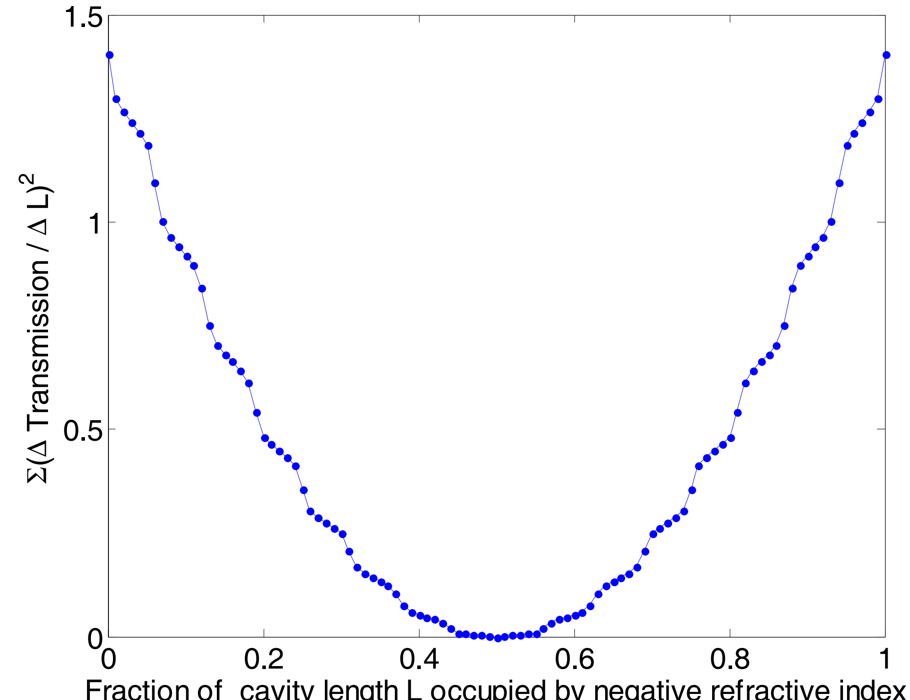


Optimization of 1D Bragg resonator with negative index material as a new degree of freedom

- ***Symmetric Optimization***
  - **Refractive index in resonator cavity is +/- 1.43**
  - **Optimum at 0.5 fraction negative refractive index material in cavity of length L**
  - **Search space highly dependent on cost functional**
  - **Global measures strongly influenced by sample space, highly irregular**
  - **Sum of local measures more robust**
  - **1D search space at a single frequency λ_0**



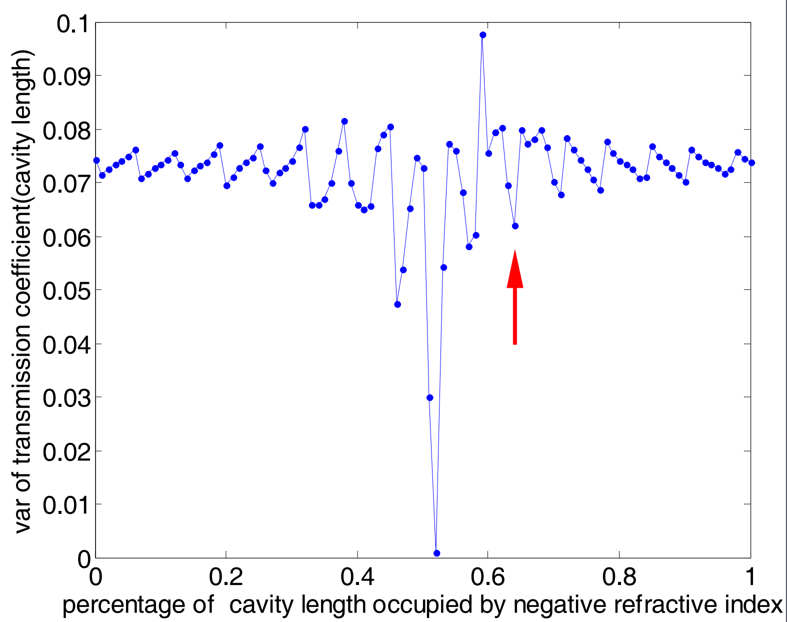
$$\text{cost1} = \sum_{n=1}^{N} \left(Transmission\left({L_i}/{\lambda_0} \right) - \frac{1}{N} \sum_{n=1}^{N} Transmission\left({L_i}/{\lambda_0} \right) \right)^2$$



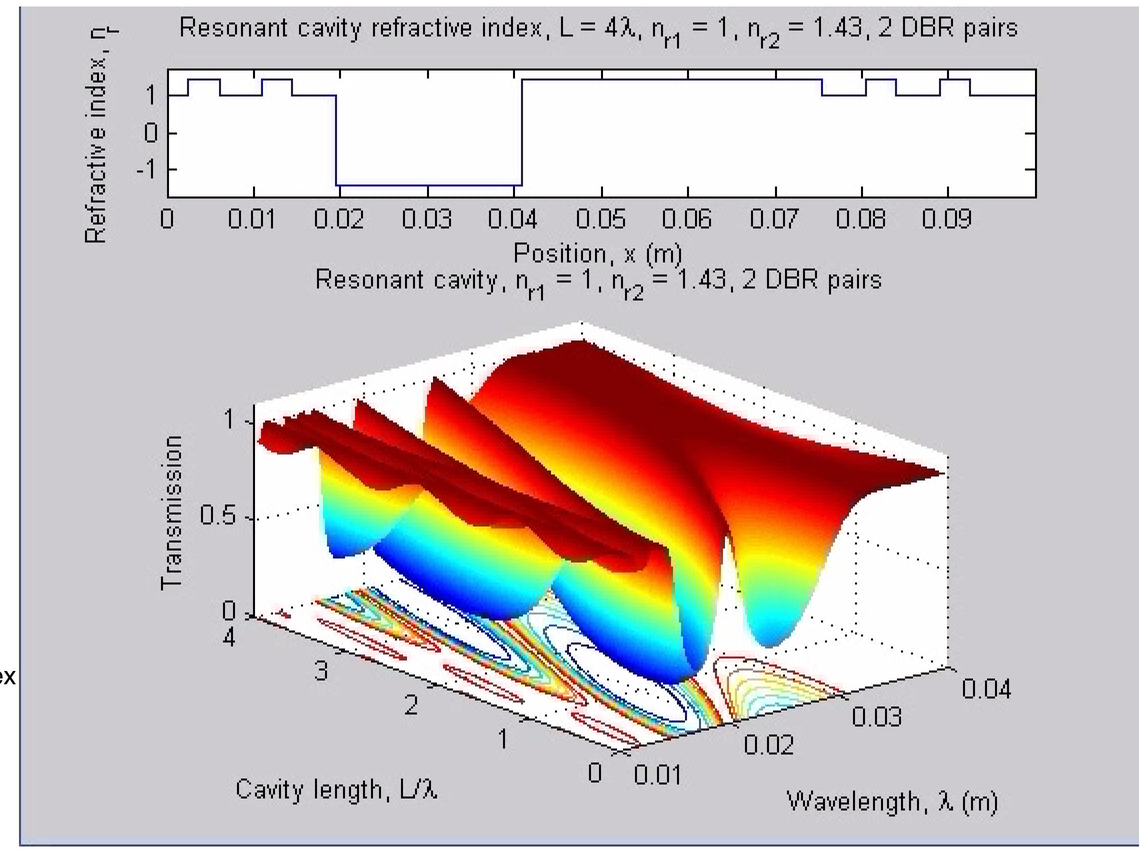
$$\text{cost2} = \sum_{n=1}^{N} \left(Transmission\left({L_{i+1}}/{\lambda_0} \right) - Transmission\left({L_i}/{\lambda_0} \right) \right)^2$$

Optimization of 1D Bragg resonator frustrated by choice of cost function

- ***Asymmetric Optimization***
  - **Refractive index in resonator cavity is +1.43 and -1.33**
    - **Optimum not at 0.5 fraction negative refractive index material in resonance cavity**
  - **Optimization frustrated by choice of cost function**
  - **Optimization routine: Matlab's fminbnd**

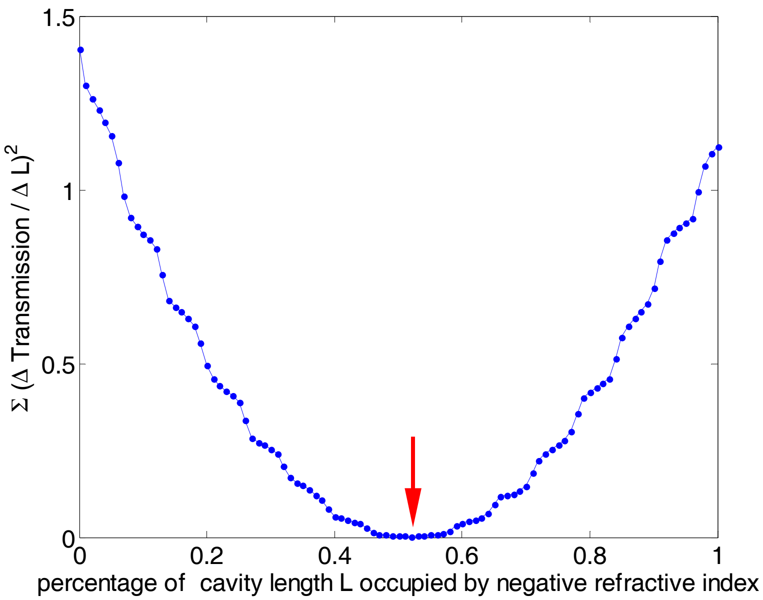


$$\text{cost1} = \sum_{n=1}^{N}\left(Transmission\left({}^{L_i}\!/_{\lambda_0} \right) - \frac{1}{N}\sum_{n=1}^{N} Transmission\left({}^{L_i}\!/_{\lambda_0} \right) \right)^2$$



Optimization of 1D Bragg resonator due to correct choice of cost function

- ***Asymmetric Optimization***
  - **Refractive index in resonator cavity is +1.43 and -1.33**
  - **Appropriate choice of cost function produces an almost parabolic search space**
  - **Optimization routine: Matlab's fminbnd**



$$\text{cost2} = \sum_{n=1}^{N}\left(Transmission\left(\frac{L_{i+1}}{\lambda_0}\right) - Transmission\left(\frac{L_i}{\lambda_0}\right)\right)^2$$