Resonant modes and laser spectrum of microdisk lasers

N. C. Frateschi and A. F. J. Levi
Department of Electrical Engineering-Electrophysics, University of Southern California, Los Angeles, California 90089-1112

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A theory for quantitative analysis of microdisk laser emission spectra is presented. Conformal mapping is used to determine the radial and azimuthal eigenvalues and eigenvectors corresponding to leaky optical modes in the disk. The results are compared with experimental data obtained from a 0.8 μm radius InGaAs/InGaAsP quantum well microdisk laser. © *1995 American Institute of Physics.*

New semiconductor microdisk[1,2] and microcylinder[3] resonant cavities have been studied with measured lasing emission wavelength at $\lambda=1550$ nm,[4] $\lambda=980$ nm,[3] and $\lambda=510$ nm.[5] Position in a cylinder is specified by natural axial coordinate z, radial coordinate r, and azimuthal angle θ. Isolated short cylinders are disks of thickness L. The design and optimization of a semiconductor microdisk laser is critically dependent on the Q of resonant optical modes as well as the spectral and spatial overlap of these modes with the active medium.

Microdisk lasers typically consist of a quantum well active region which can exhibit optical gain at, for example, $\lambda=1550$ nm. For such devices disk radius 0.5 $\mu\text{m}<R<10$ μm and thickness 0.05 $\mu\text{m}<L<0.3$ μm. Because of high optical confinement due to the air/semiconductor interface, in essence device models involve solving for the optical field $\psi(r,\theta)$ in the two-dimensional transverse direction for a medium with refractive index $n=n_{\text{eff}}$. The Helmholtz equation for optical field is separable in r and θ so that $\psi(r,\theta)=R(r)e^{iZ\theta}$ and we may write

$$r^2 \frac{d^2}{dr^2} R(r) + r \frac{d}{dr} R(r) - (k^2 r^2 + Z^2) R(r) = 0$$

and

$$\frac{d^2}{d\theta^2} \Theta(\theta) - Z^2 \Theta(\theta) = 0,$$

where $k=n_{\text{eff}}\omega/c$. Z is, in general, a complex constant. Two polarizations can be studied with the TE(TM) mode of the slab waveguide having the magnetic (electric) field in the $\hat{z}$ direction $E_z(r,\theta)[H_z(r,\theta)]$ with $n_{\text{eff}}=n_{\text{eff}}^{\text{TE(TM)}}$.

One approach to simplify the problem is to assume that optical resonances may be approximated by the whispering gallery modes (WGM) which are obtained by applying the boundary condition $\psi_{\text{in}}(R,\theta)=0$. In this situation $\psi_{\text{in}}(r,\theta)=A_M J_M(r n_{\text{eff}}\omega_{M,N}/c)e^{iM\theta}$, where J_M are Bessel functions of integer order $Z\equiv M=0, \pm1, \pm2, \pm3,...$ and A_M is a normalization constant. The boundary condition results in resonance frequencies $\omega_{M,N}=x_M^N c/n_{\text{eff}}R$, where x_M^N is the Nth zero of $J_M(r)$ and $N=1$ for WGMs. One may show that the instantaneous Poyting vector is of the form

$$\mathbf{k}_M = k_\theta(r)\cos^2(M\theta)\hat{\theta} + k_r(r)\sin(2M\theta)\hat{\mathbf{r}}$$

with propagation in the $\hat{\theta}$ direction (clockwise or counterclockwise depending on the sign of M) and $2M$ symmetrical mirror reflections with respect to the radial direction. The time average energy flux is given by $\mathbf{S}\propto(c/\mu\omega)M\hat{\theta}$ so that no optical energy escapes the disk in the radial direction.

A physically more reasonable solution is obtained by assuming a complex number $Z=M+i\alpha R$ to be the eigenvalue for the Helmholtz equation. This allows ψ to have exponential decay in the azimuthal direction and Bessel-type functions of "complex order" in the radial direction that lead to radial energy flux. Nevertheless, for high order M's and $N=1$ we anticipate a small radial flux of energy so α is very small. In this limit WGM behavior is a good approximation. However, since in these modes no energy leaves the cavity, radiation losses may not be calculated directly. For small disks, M is small since the resonance wavelengths cannot be smaller than the wavelength in the material ($x_M^N \leqslant 2\pi R n_{\text{eff}}^2/\lambda$). Therefore, in this situation, physically meaningful solutions depart considerably from the WGM picture. This letter presents results of using conformal mapping to obtain solutions for the resonant modes and respective losses in small optically transparent disks for which low M values are important.

The first step in the exact calculation is to follow the approach used by Heiblum and Harris to calculate loss in curved optical waveguides.[6] In this work a conformal transformation $u+i\nu=f(r,\theta)=R\ \ln[re^{i\theta/R}]$ is applied to the two-dimensioned Helmholtz equation. The problem is transformed into an asymmetric slab waveguide in the $\hat{\mathbf{v}}$ direction with a varying index of refraction profile $n(u)=n_{\text{eff}}e^{u/R}$ for $r\leqslant R$ and $n(u)=e^{u/R}$ for $r>R$ as illustrated in Fig. 1. Modes propagate according to $f(u,\nu)=U(u)e^{i(\beta+i\alpha)}$. For the microdisk resonator $Z=M+i\alpha R$ gives $\Psi(r,\theta)=F(r)e^{iM\theta}e^{-\alpha\theta}$ in real space and $\Omega(u,\nu)=H(u)e^{iM/R\theta}e^{-\alpha\theta/R}$ in the transformed space. That is, a wave propagating in the $\hat{\mathbf{v}}$ direction with a known propagation constant $k_\nu=M/R$ with M integer to guarantee a stationary solution in the $\hat{\theta}$ direction and a propagation loss α. In the $\hat{\mathbf{u}}$ direction

$$\frac{d^2H(u)}{du^2} + \frac{\omega^2}{c^2}\eta^2(u)H(u) = 0,$$

where plane waves in each infinitesimal slice δu propagate in the $\pm\hat{\mathbf{u}}$ direction through an index of refraction

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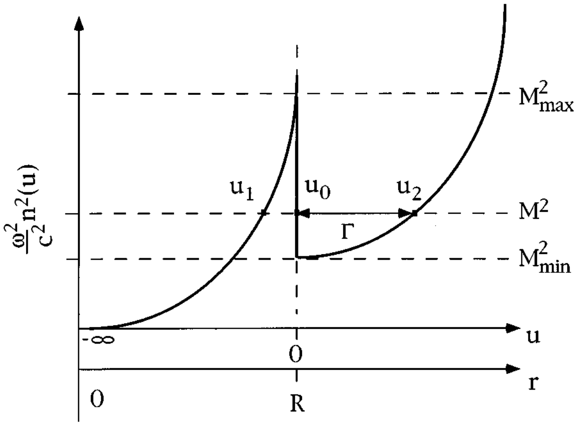

FIG. 1. Index of refraction profile for the slab waveguide in the transformed space (u,ν). The reflection points (u_0,u_1,u_2) are shown for a mode with $M_{\rm Min} \leqslant M \leqslant M_{\rm Max}$.

$$\eta(u)=\sqrt{n^2(u)-(c/\omega(M/R+i\alpha))^2}.$$

These waves change phase by $\delta\phi=(\omega/c)\,\eta(u)\,\delta u$ in the medium and are reflected at the discontinuities of $\eta(u)$. For $\alpha \ll M/R$ reflections occur at the roots of $n(u)=(c/\omega)M/R$, $u_1=R\,\ln[(c/\omega)M/Rn_{\rm eff}]$ for $u<0$ and $u_2=R\,\ln[(c/\omega)M/R]$ for $u>0$ and at the physical interface at $u_0=0$. Figure 1 shows these reflection points for a given *M*, note that u_1 and u_2 are metal-type reflections while u_0 is a dielectric-type reflection. A stationary solution in *u* will require a round-trip phase change $N2\pi$ $N=1,2,3,...$ between u_1 and u_0. For u_1 to exist $M<M_{\rm max}=2\pi Rn_{\rm eff}/\lambda$ must be satisfied. At u_0 the phase change depends on phase response from the combined dielectric- and metal-type reflections that occur at u_0, the segment Γ, and u_2. Also it depends on the polarization since for the TM(TE) slab modes $\boldsymbol{\nabla}_u H(u)[\boldsymbol{\nabla}_u H(u)/\varepsilon]$ is continuous. If these reflections are in phase, high reflectivity results and a quasiconfined stationary mode exists. The requirement on round-trip phase and constructive reflection at the $u_0-\Gamma-u_2$ mirror combination result in two equations involving α and ω for a given *M* and *N*. A quasiconfined stationary mode *M* with round-trip phase $N2\pi$ resonates with frequency $\omega_{M,N}$, loss $\alpha_{M,N}$, and a very fast optical feedback time on the scale of $2\pi/\omega_{M,N}$. We also note that when u_2 does not exist $(M<M_{\rm min}=2\pi R/\lambda)$ stationary (but not quasiconfined) states are allowed since light leaving the disk only sees a low reflectivity dielectric interface in a situation physically analogous to below the critical angle ϕ_c incidence. We expect, therefore, spectral lines with cavity $Q=M/\alpha_{M,N}R$ to occur within the range of non-quasi-confined spontaneous emission.

Figure 2 shows measured spectra for a microdisk with $R=0.8$ μm and $L=0.18$ μm. The medium has an average refractive index $n=3.456+0.333(\hbar\omega-0.74$ eV).[4] Emission peaks at $\lambda_{5,1}=1542$ nm and $\lambda_{4,1}=1690$ nm are observed in a spontaneous emission background ranging from $\lambda=1300$ nm to $\lambda=1800$ nm. To calculate the spectra for this structure we fit the calculated effective index dispersion $n_{\rm eff}=n_{\rm eff}^{\rm TE}=1.494+1.427\hbar\omega$. We have neglected TM emission since $n_{\rm eff}^{\rm TM}$ is too small to allow resonances within the spontaneous emission range. For this $n_{\rm eff}$ and the wavelength range of interest, $3<M<9$. Figure 3 shows the calculated spectral lines for this disk where modes with $Q>0.2$ were considered. The cavity *Q* increases exponentially with *M* and we observe that it reduces rapidly with *N*. The modes (5,1) and (4,1) match very well the measured resonances shown in Fig. 2, where a combination of higher *Q* and greater overlap

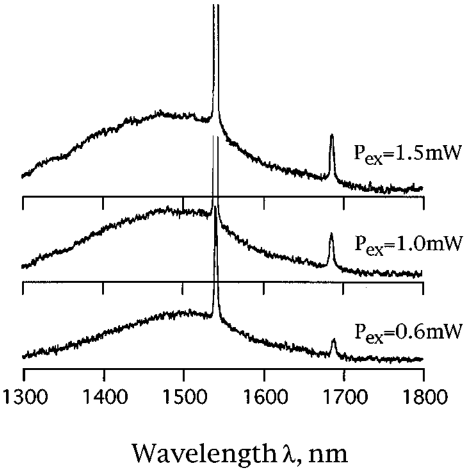

FIG. 2. Room-temperature photoluminescence spectra of $R=0.8$ μm radius microdisk laser. An AlGaAs/GaAs laser diode provides $\lambda=0.85$ μm wavelength power for the optical pump. $P_{\rm ex}$ is the incident excitation power (Ref. 4).

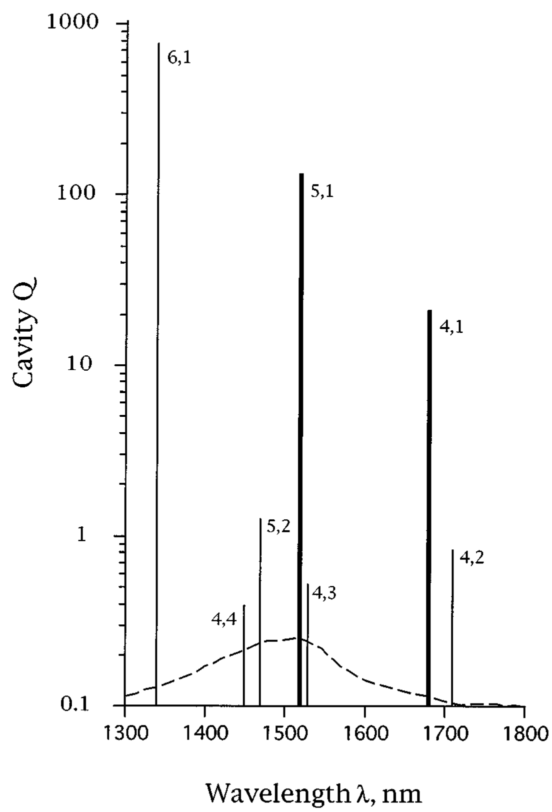

FIG. 3. Calculated spectral lines with respective cavity *Q* for the resonant modes in the $R=0.8$ μm radius microdisk laser of Fig. 2. The broken line represents the experimentally observed spontaneous emission.

with the spontaneous emission lead to the dominating mode at $\lambda_{5,1}=1540$ nm. The highest Q mode in this range (6,1) is not seen in the spectra because, unlike our model, the semiconductor is strongly absorbing at this wavelength. $M=7$, 8, 9 with higher Q are not depicted because for these resonances $\lambda<1300$ nm.

In summary, conformal mapping is used to determine the radial and azimuthal eigenvalues and eigenvectors of leaky optical modes present in dielectric microdisks. Remarkably, our model, which describes resonances in an optically transparent medium, appears to apply equally well to semiconductor microdisk lasers. Agreement with experimental results is very good even though gain and loss vary considerably over the wavelength range of spontaneous emission in the device.

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